Space-Time in the Sachdev-Ye-Kitaev Model

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OVERVIEW

1 Dimensional : Sachdev- Ye-Kitaev model

- Sachdev-Ye '93' : Georges, Parcullet, Sachdev 01': Entropy
  Sachdev ['10']: conjectured an AdS_2 Duality.
  Kitaev ['15':]: Majorana version (SYK model) Chaos,BH

Polchinski+Rosenhaus : Maldacena and Stanford:06

A.J, Suzuki & Yoon / hep-th:1603.06; hep-th:1608.07567

Talk:Questions : Emergent: 2D(3D) Space-Time/ Gravity/Matter

[S . Das , AJ, Kenta Suzuki , A.Gosch, J.Yoon,R de Mello,AJ1712.:1702.725,+to appear]

The Model

- After the disorder average [Sachdev 'et al] :

$$\langle Z^n \rangle_J = \int \mathcal{D}\chi_i \, \mathrm{T} \exp \Bigg[\frac{1}{2} \int dt \sum_{i=1}^{N} \sum_{a=1}^{n} \chi_i^a \partial_t \chi_i^a + \frac{J^2}{8N^3} \int dt_1 dt_2 \sum_{a,b=1}^{n} \left(\sum_{i=1}^{N} \chi_i^a(t_1) \chi_i^b(t_2) \right)^4 \Bigg] .$$

- Vectorial O(N) symmetry

SYK Model = O(N) Vector Model

- Replica symmetric , no spin-glass phase
- O(N) Invariant: Governed by bi-locals

$$\sum_{i=1}^{N} \chi_i^a(t_1)\chi_i^b(t_2) \;=\; \delta^{ab}\,\Psi(t_1,t_2)\,.$$

- And the collective Action :

$$S_{\rm col} \;=\; \frac{N}{2}\int dt\,\partial_t\Psi(t,t')\Big|_{t'\to t} \\ \;+\; \frac{N}{2}\mathrm{Tr}(\log\Psi) \;-\; \frac{J^2N}{8}\int dt_1dt_2\,[\Psi(t_1,t_2)]^4\,.$$

Large N (ctnd.)

- Partition Function $$\mathcal{Z} = \int [\mathcal{D}\Psi]\,\mu[\Psi]\,e^{-S_{\rm col}}$$
- Measure:FP ghosts

:

$$S_{\rm col} = \frac{N}{2}\mathrm{Tr}\left(\partial \star \Psi\right) - \frac{J^2 N}{8}\underbrace{\int dt_1 dt_2\, [\Psi(t_1,t_2)]^4 + \frac{N}{2}\mathrm{Tr}\left(\log \Psi\right)}_{S_c[\Psi]}$$

- Equivalent to two-field representation
- Critical Theory (J →∞) described by $S_c[\Psi]$

Conformal Limit

- IR: strong coupling limit

$$S_{\rm col} = \frac{N}{2}\mathrm{Tr}(\log\Psi) - \frac{J^2N}{8}\int dt_1dt_2\,[\Psi(t_1,t_2)]^4\,.$$

- with re-parametrization symmetry:

$$(t_1,t_2)\to(f(t_1),f(t_2))$$

$$\Psi(t_1,t_2) \to \left|\partial_1 f(t_1)\partial_2 f(t_2)\right|^{\frac{1}{4}}\Psi(f(t_1),f(t_2))\,.$$

[Kitaev '15, earlier in spin glass models..]

- Explicit and Spontaneous breaking of symmetry;

[Maldacena and Stanford '16] NCFT/NearAdS

Schwarzian Action

- Action of a Collective Coordinate f(t):

$$S[f] = -\frac{N}{2}\int dt_1 dt_2\, \Psi_{0,f}(t_1,t_2)\delta'(t_{12})$$

- The source in IR:

$$\delta'(t_{12}) \Rightarrow Q_s(t_1,t_2) \propto J^2 \frac{\mathrm{sgn}(t_{12})}{(J|t_{12}|)^{2-\frac{2}{q}+2s}} + \cdots$$

- Limit $s \to \frac{1}{2}$ corresponds to the leading order of $1/J$ which leads to the Schwarzian Action [AJ & K Suzuki '16]
- Similar evaluation was recently given by [Kitaev & Suh '17]

*The Propagator

- Bi-local Quadratic Action:

$$S_{(2)} = -\frac{1}{2}\int \prod_{i=1}^{4} dt_i \; \eta(t_1,t_2)\,\mathcal{K}(t_1,t_2;t_3,t_4)\,\eta(t_3,t_4)\,,$$

with four-point vertex

$$\begin{aligned}&\mathcal{K}(t_1,t_2;t_3,t_4)\\ =\;& \frac{1}{2}\Big[\psi_0^{-1}(t_4,t_1)\psi_0^{-1}(t_2,t_3) \,+\, 3J^2\delta(t_{13})\delta(t_{24})\psi_0(t_1,t_2)\psi_0(t_3,t_4)\Big]\\ &-(t_3\leftrightarrow t_4)\,.\end{aligned}$$

a two-point vertex for the bi-local field.

Result [Polchinski & Rosenhaus '16]

$$\mathcal{D}(t,z;t',z') = -\,\mathrm{sgn}(zz')\,\frac{4}{\sqrt{\pi}J}\sum_{m=1}^{\infty}\int_{-\infty}^{\infty} dw\,\frac{e^{-iw(t-t')}}{\sin(\pi p_m)}\,\frac{p_m^2}{p_m^2+(3/2)^2}$$
$$\times\left[J_{-p_m}(|wz|) + \frac{p_m+\frac{3}{2}}{p_m-\frac{3}{2}}J_{p_m}(|wz|)\right] J_{p_m}($$

Infinite sequence of massive : p_m states: physical meaning of these states

Meaning of Wave-functions? De Sitter or Anti de Sitter ?

Signature of Space-time: Euclidean Schwarzian vs Lorentzian matter

Matter : 3D Interpretation

- Propagator: pole at p_m, AdS_2 type contribution:

$$S_m^{\text{eff}} = \frac{1}{2}\int \sqrt{|g|}\, dx^2 \left[-g^{\mu\nu}\partial_\mu\phi_m\partial_\nu\phi_m - M_m^2\phi_m^2\right] ,$$

Where masses $$M_m^2 = p_m^2 - \frac{1}{4} .$$

are given by the quantization condition

$$-\frac{2}{3}\, p_m = \tan\left(\frac{\pi p_m}{2}\right) .$$

much like a sequence of higher (spin)states: quantization condition?.

HIGHER DERIVATIVE ADS_2

- Infinite sequence of poles:

$$S \propto \Psi(t,z) \prod_{m=1}^{\infty} \left(\Box_{\mathrm{AdS}_2} - M_m^2 \right) \Phi(t,z) \, ,$$

With

$$M_m^2 = p_m^2 - \frac{1}{4} \, .$$

An Infinitely Non-polynomial Action/

PLUS: a 1d Schwarzian coordinate f(t).

Cont. 3D Gravity

- Confining the extra dimension: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\,,$

With a delta potential at y=0 gives the: quantization

$$-\frac{2}{3}\,p_m = \tan\left(\frac{\pi p_m}{2}\right)\,.$$

Propagator at $y = y' = 0$

$$G \propto \sum_{m=1}^{\infty}\int_{-\infty}^{\infty} d\omega\,\frac{e^{-i\omega(t-t')}}{\sin(\pi p_m)}\,\frac{p_m^2}{p_m^2+(3/2)^2}$$

:

$$\times\left[J_{-p_m}(|\omega z|)+\left(\frac{p_m+\frac{3}{2}}{p_m-\frac{3}{2}}\right)J_{p_m}(|\omega z|)\right]J_{p_m}(|\omega z'|)\,.$$

Question of Space-Time

- Lorentzian signature for Euclidean SYK comes from bi-local: $$t \equiv \frac{t_1 + t_2}{2}, \qquad \eta \equiv \frac{t_1 - t_2}{2},$$

the bi-local SL(2, R) Casimir becomes

$$C_{1+2} = -\eta^2(-\partial_\eta^2 + \partial_t^2),$$

which is the Laplacian of Lorentzian dS_2 space

$$ds^2 = \frac{-d\eta^2 + dt^2}{\eta^2}.$$

WaveFunctions

- In fact, the eigenfunctions

$$u_{\nu,\omega}(\eta,t) \;=\; \mathrm{sgn}(\eta)\,|\eta|^{\frac{1}{2}}\,e^{i\omega t}\,Z_\nu(|\omega\eta|)\,,$$

are particular choice of alpha-vacuum wave functions in Lorentzian dS_2 space with

$$\alpha \;=\; i\pi\left(\nu+\tfrac{1}{2}\right) \;=\; i\pi h\,.$$

- In general, alpha-vacuum wave functions are

$$\phi^\alpha_\omega(\eta) \;=\; N_\alpha\,\eta^{\frac{1}{2}}\Big[H^{(2)}_\nu(|\omega|\eta) \;+\; e^\alpha H^{(1)}_\nu(|\omega|\eta)\Big]\,.$$

But

- Neither, dS_2 nor AdS_2 :can be right for the spacetime:

  Problem of "i "
- A Lorentzian Gravity

$$Z = \int \mathcal{D}\Phi_m \exp\left[i\left(S_{\rm grav}[\Phi] + S_{\rm matter}[\Phi]\right)\right],$$

While the action is real for the SYK

$$Z = \int \mathcal{D}\Psi\, e^{-S_{\rm col}[\Psi]}.$$

- Mismatch of "i" : for all n-point functions

Resolved in: S Das et al:arXiv:1712.02725

Vectorial Holography

- d=1 : SYK
- d=2, 3 : (Large N) Vector Model / Higher Spin Gravity
- d=3 : *

$$\mathcal{L} = (\partial\vec{\phi})\cdot(\partial\vec{\phi}) + \frac{\lambda}{2N}(\vec{\phi}\cdot\vec{\phi})^2$$

UV : $\lambda = 0$ IR : $\lambda = \infty$ Wilson-Fisher

[Klebanov & Polyakov '02] [Sezgin & Sundell '02] [Giombi & Yin '09]

- d=2 : AdS_3 HS / Minimal Model [Gaberdiel & Gopakumar '10]
- * Vasiliev Hgher Spin Theory in AdS

Bi-local (Re)Construction

$$\Psi(x_1, x_2) = \frac{1}{N} \sum_{i=1}^{N} \phi_i(x_1)\phi_i(x_2)$$

CFT_d / AdS_{d+1} + Spin (S^{d-1})

- AJ +Sumit Das (2006),de Mello Koch, K.Jin, J.Rodrigues, J.Yoon(2011-)

Bi-local Field: $\Psi(x_1^\mu, x_2^\mu) \Leftrightarrow H(x^\mu, z; S)$

$$d + d = (d+1) + (d-1)$$

- One to One Map to Higher Spins in AdS_(d+1) .

AdS_3 /CFT_2 :SO(2,2)=SO(2,1)x SO(2,1)

- Large N Action : Laplacian

$$\widehat{\mathcal{L}}_{\mathrm{bi}} \approx C_4 + \frac{1}{4} C_2^2$$
$$\approx \frac{1}{4} |x_1 - x_2|^4 \frac{\partial}{\partial x_1} \cdot \frac{\partial}{\partial x_1} \frac{\partial}{\partial x_2} \cdot \frac{\partial}{\partial x_2}$$

In terms of Casimirs

$$C_2 \equiv \frac{1}{2} L_{AB} L^{AB}$$
$$C_4 \equiv \frac{1}{4} L_A{}^B L_B{}^C L_C{}^D L_D{}^A - \frac{1}{2} C_2^2$$

'Kinematic space" X :does not produce this.

Eigenvalue Problem

- Eigenvalue Problem of Bi-local Laplacian:

$$\widehat{\mathcal{L}}_{\text{bi}}\,\psi_{h,s}(x_1,x_2)\;=\;\lambda(h,s)\,\psi_{h,s}(x_1,x_2)$$

- The eigenfunctions are conformal blocks/ spinning 3-pt funcs:

$$\psi_{h,s}(\vec{x}_1,\vec{x}_2;\vec{x}_3)\;=\;\frac{(V^{\mu}z_{\mu})^{s}}{|x_1-x_3|^{h+s}|x_2-x_3|^{h+s}|x_1-x_2|^{2\Delta+s-h}}$$

With eigenvalues:

$$\lambda(h,s)\;=\;\frac{1}{4}\,(h^2-s^2)^2$$

Higher Spin Theory:s=0,1,2,3..

- AdS{d+1} HS fields

$$(\Box - m^2)h_{\mu_1\cdots\mu_s} + s\,\nabla_{(\mu_1}\nabla^{\nu}h_{\mu_2\cdots\mu_s)\nu} - \frac{s(s-1)}{2(d+2s-3)}\,g_{(\mu_1\mu_2}\nabla^{\nu_1}\nabla^{\nu_2}h_{\mu_3\cdots\mu_s)\nu_1\nu_2} = 0$$

- Can be packaged into a 2x(d+1) field of all spins:

$$H(X;Y) \equiv \sum_s H_{A_1\cdots A_s}(X)\,Y^{A_1}\ldots Y^{A_s}$$

- Reduction to physical degrees gives a d+d realization

Gauge Reduction for HS Theory

- Physical Polarizations :obtained by solving the tracelessness & de Donder gauge conditions:

$$
\begin{aligned}
\eta^{\hat{\mu}_1\hat{\mu}_2} H_{\hat{\mu}_1\hat{\mu}_2\cdots\hat{\mu}_s} &= 0 \\
\left(\partial_z - \frac{1}{z}\right) H_{z\hat{\mu}_2\cdots\hat{\mu}_x} + \partial_\mu H_{\mu\hat{\mu}_2\cdots\hat{\mu}_s} &= 0
\end{aligned}
$$

For AdS3 results in

$$
H(x^i, z, \theta) = \sum_{s=1}^{\infty} e^{\pm is\theta} H_{(\pm s)}(x^i, z)
$$

Physical HS :AdS3+S1

$$H(x^i, z, \theta) = \sum_{s=1}^{\infty} e^{\pm is\theta} H_{(\pm s)}(x^i, z)$$

- SO(2,2) symmetry generators

$$L_-^{\mathrm{AdS}_3\times\mathrm{S}^1} = -ip^+$$

$$L_0^{\mathrm{AdS}_3\times\mathrm{S}^1} = p^+x^- - \frac{1}{2}zp^z$$

$$L_+^{\mathrm{AdS}_3\times\mathrm{S}^1} = -ip^+(x^-)^2 + ix^-zp^z - iz^2p^- + \frac{\sqrt{4p^+p^- - (p^z)^2}}{2p^+}izp^\theta - i\frac{(p^\theta)^2}{4p^+}$$

In terms of which the HS AdS Laplacian

$$\mathcal{L}_{\mathrm{AdS}_3\times S^1} = C_4 + \frac{1}{4}C_2^2$$

SO(2,2) Generators

Bi-local form

$$L_{-}^{\mathrm{bi}} = -i(k_1^+ + k_2^+)$$

$$L_0^{\mathrm{bi}} = k_1^+ \frac{\partial}{\partial k_1^+} + k_2^+ \frac{\partial}{\partial k_2^+}$$

$$L_+^{\mathrm{bi}} = i\left[k_1^+ \left(\frac{\partial}{\partial k_1^+}\right)^2 + k_2^+ \left(\frac{\partial}{\partial k_2^+}\right)^2\right]$$

To be mapped into the AdS3xS1 representation

$$L_{-}^{\mathrm{AdS_3\times S^1}} = -ip^+$$

$$L_0^{\mathrm{AdS_3\times S^1}} = p^+x^- - \frac{1}{2}zp^z$$

$$L_+^{\mathrm{AdS_3\times S^1}} = -ip^+(x^-)^2 + ix^-zp^z - iz^2p^- + \frac{\sqrt{4p^+p^- - (p^z)^2}}{2p^+} izp^\theta - i$$

Momentum Map:

- Momentum space change of variables:

$$
\begin{aligned}
\vec{p} &= \vec{k}_1 + \vec{k}_2 \\
p_z &= 2\sqrt{k_1^+ k_1^-} - 2\sqrt{k_2^+ k_2^-} \\
\theta &= \frac{1}{2}\left(\arcsin\frac{k^+}{p^+} - \arcsin\frac{k^-}{p^-}\right)
\end{aligned}
$$

- Fourier transforms give the space-time kernels for maps between Bi-local and Ads spaces

$$
\begin{aligned}
\overline{\Psi}(x_1^i, x_2^i) &= \mathcal{M}(x_1^i, x_2^i | x^i, z, S)\, H(x^i, z, S) \\
H(x^i, z, S) &= \mathcal{M}^{-1}(x^i, z, S | x_1^i, x_2^i)\, \overline{\Psi}(x_1^i, x_2^i)
\end{aligned}
$$

Cubic and Higher Vertices

- The cubic vertex in bi-local CFT_2:

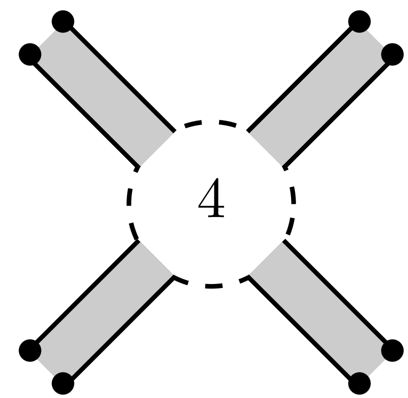
$$V^+_{(3)}(k_1, k'_1; k_2, k'_2; k_3, k'_3) \ \propto \ \delta(k^+_1 + k'^+_2)\delta(k^+_2 + k'^+_3)\delta(k^+_3 + k'^+_1)$$

is transformed under the bi-local into AdS cubic vertex

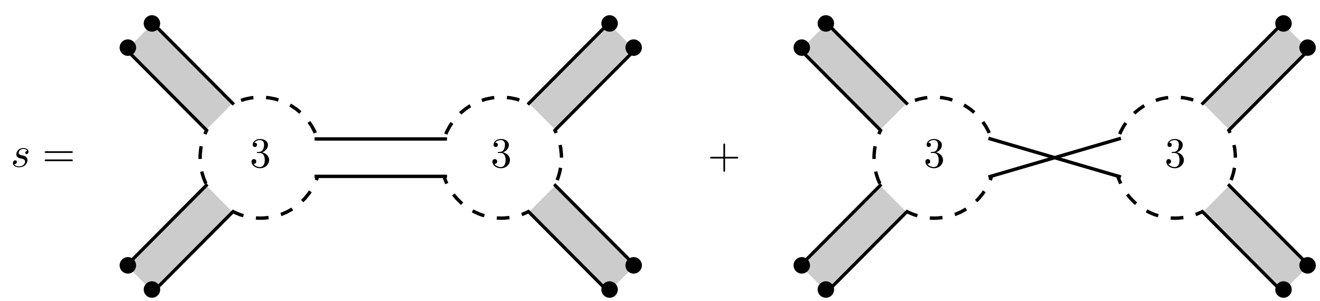
$$\begin{aligned} & V^+_{(3)}(p_1, \phi_1, \theta_1; p_2, \phi_2, \theta_2; p_3, \phi_3, \theta_3) \\ \propto\ & \delta(p^+_1 + p^+_2 + p^+_3)\, \delta\Big(p^+_2 \left(1 + \sin(\phi_2 + \theta_2)\right) + p^+_3 \left(1 - \sin(\phi_3 + \theta_3)\right)\Big) \\ & \times\ \delta\Big(p^+_1 \left(1 - \sin(\phi_1 + \theta_1)\right) + p^+_3 \left(1 + \sin(\phi_3 + \theta_3)\right)\Big) \end{aligned}$$

Bulk Feynman rules

- For tree-level 4-pt func,
  we have the contact diagram:



- And s-, t- u-channel exchange diagrams:



SYK:SO(2,1): EAdS Space-time

- Momentum space kernel is

$$\mathcal{R}(p_1, p_2; p_\tau, p_z) \, = \, \frac{\delta(p_\tau - (p_1 + p_2))}{\sqrt{p_z^2 + 4p_1p_2}} \, .$$

- Fourier transform to coordinate space leads to

$$\mathcal{R}(t_1, t_2; \tau, z) \, = \, \delta(\eta^2 - (\tau - t)^2 - z^2) \, ,$$

* the Radon transform kernel.

Between Euclidean AdS - Lorentzian dS,

- Radon transform

$$[\mathcal{R}f](\eta,t) = 2\eta \int_{t-\eta}^{t+\eta} d\tau \int_0^\infty \frac{dz}{z}\, \delta\Big(\eta^2 - (\tau - t)^2 - z^2\Big)\, f(\tau,z)\;.$$

- And the Inverse

$$\mathcal{R}^{-1}\, \underbrace{\psi_{\omega,\nu}(\eta,x)}_{\text{Bi-local wave functions}} = F(\nu)\, \underbrace{z^{\frac{1}{2}}\, e^{-i\omega\tau}\, K_\nu(|\omega| z)}_{\text{EAdS\_2 wave functions}}$$

$F(\nu)$:

$$L(\nu) \equiv (\text{Leg Factor}) = -2i\sqrt{\pi}\,\frac{\Gamma(\frac{1}{4}+\frac{\nu}{2})}{\Gamma(\frac{3}{4}+\frac{\nu}{2})}\,.$$

EAdS: Propagator and Leg factor

- The bi-local propagator with $\eta = (t_1 - t_2)/2$ and $t = (t_1 + t_2)/2$

$$G(\eta, t; \eta', t') \propto \int d\omega \Bigg[\sum_{n=0}^{\infty} \frac{4 \sin \pi\nu_n}{\widetilde{g}(\nu_n) - 1} \, \overline{\psi}^{\,*}_{\omega,\nu_n}(\eta, t) \, \overline{\psi}_{\omega,\nu_n}(\eta', t') \bigg|_{\nu_n = 2n + \frac{3}{2}} + \int_0^{\infty} dr \, \frac{\overline{\psi}^{\,*}_{\omega,\nu}(\eta, t) \, \overline{\psi}_{\omega,\nu}(\eta', t')}{\widetilde{g}(\nu) - 1} \bigg|_{\nu = ir} \Bigg] .$$

- Inverse Radon transform brings the bi-local propagator into EuclideanAdS space-time

$$G(\tau, z; \tau', z') \propto \int d\omega \Bigg[\sum_{n=0}^{\infty} \frac{4 \sin \pi\nu_n}{\widetilde{g}(\nu_n) - 1} \, |L^{-1}(\nu_n)|^2 \, \overline{\phi}^{\,*}_{\omega,\nu_n}(\tau, z) \, \overline{\phi}_{\omega,\nu_n}(\tau', z') + \int_0^{\infty} dr \, |L^{-1}(\nu)|^2 \, \frac{\overline{\phi}^{\,*}_{\omega,\nu}(\tau, z) \, \overline{\phi}_{\omega,\nu}(\tau', z')}{\widetilde{g}(\nu) - 1} \bigg|_{\nu = ir} \Bigg] .$$

Leg factors and Extra States

- After contour integral of ν, there is an appearance of extra on-shell states in the second line:

$$G(\tau, z; \tau', z') \propto \int_{-\infty}^{\infty} d\omega\, e^{-i\omega(\tau-\tau')}$$

$$\times \left\{ \sum_{m=0}^{\infty} \frac{\Gamma(\frac{3}{4}+\frac{p_m}{2})\Gamma(\frac{3}{4}-\frac{p_m}{2})}{\Gamma(\frac{1}{4}+\frac{p_m}{2})\Gamma(\frac{1}{4}-\frac{p_m}{2})} \frac{p_m}{\widetilde{g}'(p_m)} K_{p_m}(|\omega|z^>) I_{p_m}(|\omega|z^<) \right.$$

$$\left. + \sum_{n=0}^{\infty} \frac{\Gamma^2(\frac{3}{4}+\frac{\nu_n}{2})}{\Gamma^2(\frac{1}{4}+\frac{\nu_n}{2})} \left(\frac{\nu_n}{\widetilde{g}(\nu_n)-1} \right) \left[2I_{\nu_n}(|\omega|z^>) - I_{-\nu_n}(|\omega|z^>) \right] I_{\nu_n}(|\omega|z^<) \right\}$$

- For n-point functions to: Extra L factors
- This then are to be recovered by Euclidean AdS Gravity

Implication

- Leg factors: $|L^{-1}(\nu)|^2$ additional poles :discrete states.
- Analogy with c=1 matrix model/2D string duality, where the extra leg pole factors s "discrete states" in the 2D string theory.
- We suggest that these extra states appear in SYK : represent would be gauge degrees of freedom of the dual(hs) gravity .

Matrix Model/2D String(ctnd)

- Liouville background

$$\mathcal{L}_{\mathrm{Lio}} = -\partial_t^2 + \partial_\varphi^2 - \mu e^{-2\varphi}$$

- The map :from collective density to Liouville $ik = \ell = e^{-\varphi}$

$$\Phi(t,\ell) = \int_0^\infty d\tau\, e^{-\ell\sqrt{\mu}\cosh\tau}\, \eta(t,\tau)$$

- Now the propagator is

$$\left\langle \Phi(t,\varphi)\Phi(t',\varphi') \right\rangle = \int_{-\infty}^{\infty} dE \int_0^\infty dp\, \frac{p}{\sinh \pi p} \frac{\phi^*_{E,p}(t,\varphi)\phi_{E,p}(t',\varphi')}{E^2 - p^2}$$

with

$$\phi_{E,p}(t,\varphi) = \sqrt{p \sinh \pi p}\, e^{-iEt}\, K_{ip}(\sqrt{\mu}e^{-\varphi})$$

2D String:Propagator

(Moore+Seiberg 91'):evaluation

$$\left\langle \Phi(t,\varphi)\Phi(t',\varphi') \right\rangle$$

$$= -\pi \int_{-\infty}^{\infty} dE\, e^{-iE(t-t')} \left\{ \frac{\pi E}{2\sinh \pi E}\, K_{iE}(\sqrt{\mu}e^{-\varphi^<})\, I_{iE}(\sqrt{\mu}e^{-\varphi^>}) \right.$$

$$\left. + \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{E^2+n^2}\, K_n(\sqrt{\mu}e^{-\varphi^<})\, I_n(\sqrt{\mu}e^{-\varphi^>}) \right.$$

- (i) Tachyon: $|p| = E$
- (ii) Discrete Modes: $p = in \qquad (n \in \mathbb{Z}^+)$

Close parallel with SYK

Both have :

- Discrete states
- And : Matter

$c=1$	3D SYK
$ie^{-\varphi}$	z
t	y
ip	ν
E	k
$\sqrt{\mu}$	$\lvert\omega\rvert$

- Discrete states found as solutions of 2d Dilaton Gravity and are present in AdS also/

- Higher n-point functions : discrete states are exchanged in scattering processes:

CONCLUSIONS

- Euclidean SYK can be represented as EucledianAdS
- Map: (momentum space) from Bi-local to theEAdS picture :with Leg factors
- Exhibit extra: Discrete states in addition to Bi-local matter
- Such States appear in AdS_2 (H S) Gravity
- Role in finding the Gravity dual of SYK